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**Q1**

-There are two semi-infinite plane mirrors inclined physically at a non-zero angle with the inner surfaces being reflective.

Prove that all lines of incident/reflected rays are tangential to a particular circle for any given incident ray being incident on a reflective side. Assume that the incident ray lies on one of the normal planes to the mirrors.
- There's a cone of an arbitrary base with large enough length.

The inner surface polished (Outer surface is absorbing in nature) and the apex is fixed to a point. The cone is being rotated around the apex at an angular speed $\omega$ around the vertical axis and assume that a large part of the inside is visible horizontally. A fixed horizontal ray is projected from outside towards the cone (which often falls inside of it), prove that all the lines of incident ray/reflected rays at all instants lie tangential to a particular sphere.

Try guessing the radius of the sphere with the parameters you observe.

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**Q2** A regular tetrahedral massless frame whose side length is physically variable (with the constraint of the tetrahedron being regular) is dipped in a soap solution of surface tension $T$, taken outside and allowed to settle after a little wiggle.

The soap film is formed such that there is no volume in space that is enclosed by any of the surfaces soap film and all the soap film surfaces are planar. You may assume the configuration of the soap film without proof.

Now 4 point charges of charge $q$ are fixed at the vertices of the tetrahedron.

The system now sets into motion with the shape and nature of soap film being unaltered at all times.

- Find the side length of the tetrahedron for which the system attains mechanical equilibrium.
- Find the differential equation(s) governing the side length with respect to time.
- If the amplitude of oscillations are very small, find the time period of oscillations.

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**Q3** **Newton's Law of Gravity from Kepler's Laws?**

 

- Planets in the solar system move in elliptic orbits with the sun at one of the foci.
- The line joining the sun and the planet sweeps out equal areas in equal times.
- The period of revolution ($T$) and the length of the semi-major axis ($a$) of the ellipse sit in the relation $T^2/a^3 = constant$.

Now answer the following questions:

- Starting from Newton's Law of Gravitation and Kepler's first law, derive the second and third law. It is possible to derive the first law but that is beyond the scope of this exam.
- For convenience work in the complex (Argand) plane and take the sun to be at the origin $(z = 0)$. Show that points on the ellipse may be represented by,

$$z(\theta) = \frac{a(1-\epsilon^2)}{1+\epsilon\cos\theta}\exp(i\theta) = r(\theta)e^{i\theta}$$

where $a$ is the length of the semi-major axis, $\epsilon$ is the eccentricity of the ellipse and $\theta$ is called the *true anomaly* in celestial mechanics.
- Show that Kepler's second law leads to,

$$\frac{1}{2}r^2\dot{\theta} = constant$$

where $r$ and $\theta$ are defined as in part (b) and a dot $(.)$ over a variable denotes its time derivative. What is this constant in terms of the other variables of the problem?
- Using the results of parts (b) and (c) along with Kepler's third law obtain Newton's Law of Gravitation.
- Can the above exercise truly be called a "derivation" of Newton's Law of Gravitation? State your reasons.