TESSELLATE PRESENTS

STEMS

Scholastic Test of Excellence in Mathematical Sciences

Physics Category B

Exam Date : 25th January 2020
Exam Timing : 4pm - 7pm

Rules and Regulations

Marking Scheme

1. The question paper is divided in two parts -
   Objective (10 questions) + Subjective (3 questions).
2. Each objective question is worth **2 points**.
3. Each subjective problem is worth **10 points**.
4. There is no negative marking in any section.
5. **The subjective part will be graded only if you score above a certain cut-off (to be decided later) in the objective section of the paper**.
6. **For the final score, your total score (subjective + objective) will be taken into consideration.**

Solution guidelines

1. You are **NOT** required to show your work for the objective part of the paper.
   Only tick your option choices in a tabular format drawn on a blank sheet.
   A sample is shown below.

Q. No.	(a)	(b)	(c)	(d)
1		✓		
2			✓	
3		✓		
4	✓			
5				✓

Fig. - Sample objective answer submission

2. Provide a complete solution/proof for subjective questions. The solutions must be correct, original, detailed and clearly understandable for full credit.
   Partial credit might be awarded to incomplete proofs, based on the progress made towards solving the problem.
3. Draw clear, well-labeled diagrams wherever necessary.

Miscellaneous

1. Answers should be your own and should reflect your independent thinking process.
2. Do **NOT** post the questions on any forums or discussion groups. It will result in immediate disqualification of involved candidates when caught.
3. Answers should be written clearly, in a legible way. Formal proofs are required wherever asked for. Unclear reasoning might not be awarded points, draw clear diagrams wherever necessary.
4. Sharing/discussion aimed towards solving or distribution of problems appearing in the contest while the contest is live in any kind of online platform/forum shall be considered as a failure in complying with the regulations.
5. Any form of plagiarism or failure to comply with aforementioned regulations may lead to disqualification.

Contact details - ONLY for subject related queries

- Please do not call these people for technical problems or submission inquiries.
  Only if you find an ambiguity in a question and need clarification, use these contacts.
- As our phone numbers will be busy, **we prefer WhatsApp & email queries** instead. Only call us if absolutely necessary.
- Try to solve all your submission related doubts from information in the next page ONLY. We have included all details in the next section.

Sriram Akella - 7995855349	Prasanna Venkatesan - 9791310069
Query email - stems.2020.enq.phy@gmail.com	

- **Do NOT call any number to ask if your submission has reached us.**
  If you send your submission to the right email address with mentioned details, we will receive it. We will contact all participants who fail to submit, so please be patient.

How to submit your answers

1. Write the following details **as per your registration** on the first page of your submission file/photographs -

Name	:	Your full name
School/College name	:	
Class/Year of Study	:	Class 8, Class 11, Undergrad 1st year etc.
Registered Email address	:	
Mode of S.T.E.M.S. Registration	:	Online / Through School / Other (mention details)

2. Write all your answers on sheets of paper, following all the solution guidelines. Write the page number in the top right corner of every sheet.

3. Scan your answers or take clear photographs of your response sheets. Compile them into a single PDF file or send all pictures in the right sequence.

   If you have a limited file size issue when sending your submission, make a new Google Drive folder with title '**Your Name - Physics B submission**'.

4. Send the submission file or Google Drive link from your registered email -

Submission email address	:	**stems.2020.phy.b@gmail.com**
Subject of email	:	Physics Category B Submission - STEMS
Submission deadline	:	25th January 2020 - 19:00

Good luck, happy problem-solving!

Objective Questions

For **Problems 1-10**, each problem has **four** options, namely **(a), (b), (c), (d)**, of which ***only one*** is correct, **2 point** will be awarded for correctly answering a problem, **NO** negative marks shall be awarded for wrong answers/unattempted problems.

Problem 1. *Given a particle with mass $m = k$ moving along the x-axis under the action of the force $F(x) = -kx + 2020/x^n$. The time period of small oscillations about the equilibrium point is:*

(a) $\frac{2\pi}{\sqrt{n+1}}$

(b) $2\pi\sqrt{n+1}$

(c) $\frac{2\pi}{\sqrt{n^2+1}}$

(d) $2\pi\sqrt{n^2+1}$

Problem 2. *Two concentric spherical shells of radius a and b ($a < b$) are situated in space. a-radius sphere has uniform charge density σ_1 ($\sigma_1 > 0$) and b-radius sphere has uniform charge density σ_2 ($\sigma_2 < 0$). The energy of this configuration is*

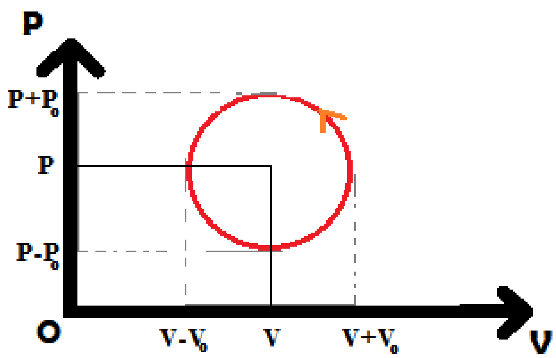
(a) $\dfrac{1}{2\pi\epsilon_0}[a^3{\sigma_1}^2 + b^3{\sigma_2}^2 - 2a^2b\sigma_1\sigma_2]$

(b) $\dfrac{1}{4\pi\epsilon_0}[a^3{\sigma_1}^2 + b^3{\sigma_2}^2 - 2a^2b\sigma_1\sigma_2]$

(c) $\dfrac{2\pi}{\epsilon_0}[a^3{\sigma_1}^2 + b^3{\sigma_2}^2 - 2a^2b\sigma_1\sigma_2]$

(d) $\dfrac{4\pi}{\epsilon_0}[a^3{\sigma_1}^2 + b^3{\sigma_2}^2 - 2a^2b\sigma_1\sigma_2]$

Problem 3. *Consider one mole of a monoatomic ideal gas. The gas undergoes a cyclic process given by an circle(anti-clockwise) on the P-V plane (Pressure is along the Y-axis, Volume is along the X-axis), as shown in the diagram. The work done in the process by the gas is:*



(a) πP_o^2

(b) πV_o^2

(c) $\pi P_o V_o$

(d) $2\pi P_o V_o$

Problem 4. *Atmosphere of a planet consist of a gas of molar mass μ (constant density throughout). Let mass and radius of planet be M and r respectfully. If the height of the atmosphere is $h\,(<< r)$, the temperature of the atmosphere is (R denotes gas constant)*

(a) $\dfrac{\mu GMh}{Rr^2}$

(b) $\dfrac{2\mu GMh}{Rr^2}$

(c) $\dfrac{\mu GMh}{2Rr^2}$

(d) $\dfrac{3\mu GMh}{Rr^2}$

Problem 5. *Consider a hot-air(captured from atmosphere) balloon with a fixed volume $V_H = 1.1\, m^3$ and pressure same as that of atmospheric pressure. The mass of the balloon envelope is $m_H = 0.187\, kg$. The external air temperature is $\theta_{ext} = 20\,^oC$ and the normal external air pressure is $P_{ext} = 1.013 \times 10^5\, Pa$. Under these conditions the density of air is $\rho_{air} = 1.2\, kg/m^3$. What must be the temperature (in degree Centigrade) of the air inside the balloon to make it just float? (Answer upto 2 decimal places by truncation)*

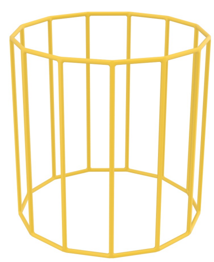
(a) 67.54

(b) 68.38

(c) 66.27

(d) 69.00

Problem 6. *Consider a thin rigid metal structure, with a shape as in the picture. We dip the structure in a soap solution of Surface Tension T. Let r_{sphere} be the radius of the truncated soap sphere emerging from the circular part of the frame. If $r_{cylinder}$ is the radius of the circular part of the structure, find $\frac{r_{sphere}}{r_{cylinder}}$:*



(a) 1

(b) 2

(c) 3

(d) 4

Problem 7. *A conducting rod of length 2l is rotating with constant angular velocity* ω *about its perpendicular bisector. A uniform magnetic field* $\bar{B}$ *exists parallel to the axis of rotation. The emf induced between the two ends of the rod is:*

(a) $\frac{1}{2}B\omega l^2$

(b) $B\omega l^2$

(c) 0

(d) *None of these*

Problem 8. N *men, each with mass* m*, stand on a flat boat of mass* M *in a still lake. Suppose all the men jump off at the same time with a velocity* $\overline{u}$ *relative to the boat, then the boat moves with a velocity* $\overline{V}_{NR}$*. Suppose the men jump off one after the other with a velocity* $\overline{u}$ *relative to the boat, then the velocity of the boat after all the men have jumped is* $\overline{V}_{NN}$*. Compute the ratio* $\frac{|\overline{V}_{NN}|}{|\overline{V}_{NR}|}$ *in the limit* $N \longrightarrow \infty$*.*

(a) $\frac{m}{M}$

(b) $\frac{M}{m}$

(c) 0

(d) ∞

Problem 9. *A source emits sound at a constant frequency and constant wavelength(*λ*) uniformly in all directions is placed at one of the focus of ellipsoid and the other focus has a point observer. The major axis of the ellipsoid is* $a = N\lambda$*, and the minor axes are of same length* $b = M\lambda$*. Find the minimum phase difference between any two wavelets received by the observer.*

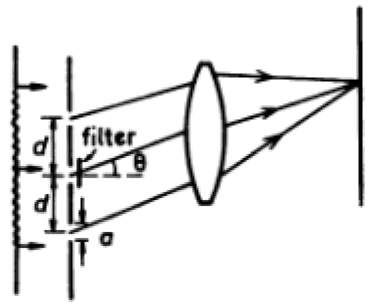
(a) $2\pi(2M - \sqrt{N^2 - M^2})$

(b) $4\pi\sqrt{M^2 - N^2}$

(c) $4\pi\sqrt{N^2 - M^2}$

(d) $2\pi(2N - \sqrt{N^2 - M^2})$

Problem 10. *A plane wave of wavelength λ is incident on a system having 3 slits of width a separated by a distance d. The middle slit is covered by a filter which introduces a 180^o phase change. Calculate the angle θ for the first interference minimum.*



(a) $\theta = \pm \sin^{-1}(\frac{\lambda}{a})$

(b) $\theta = \pm \sin^{-1}(\frac{\lambda}{d})$

(c) $\theta = \pm \sin^{-1}(\frac{\lambda}{6d})$

(d) $\theta = \pm \sin^{-1}(\frac{\lambda}{6a})$

Subjective Problems

Problem 1.

A Tessellation in 2-D is tiling whole of a plane using geometric shapes(called tiles) with no overlapping and no gaps. Such a Tessellation is called regular if all tiles are congruent to a regular polygon.

i. *Find all such possible regular polygons for which a regular tessellation is possible.[Let's call this set of polygons as set* ***P****]*

ii. *Compute the ratio of time periods (with ratios in ascending order) for a particle moving with constant speed on shapes belonging to set* ***P****. It's given that all the shapes in set* ***P*** *have the same side length.[Time period is defined as time required for the particle to complete one revolution around a closed shape]*

iii. *We replace each edge in the Tessellation with resistors of resistance R.*
Find the equivalent resistance between two adjacent points in each possible tessellation.

iv. *For a particular vertex in the regular Tessellation, consider the collection of shapes sharing that point as vertex. If we assume each edge to have a mass m and length l, find the Moment of inertia of the setup about the axis perpendicular to the plane and passing through the vertex of consideration.*

v. *For each shape in set* ***P*** *construct a corresponding prism of length l with that shape being the base such that, the prism's wall is adiabatic and prism's structure is rigid. All the bases of prisms have same circumradius. Let a given number of mole(s) of ideal gas be present in each of the prism with gases having degrees of freedom equal to the number of sides in the base of prism.*
What is the ratio of the work done by each of the gas in an arbitrary process undergone by it between two any two possible fixed states?

Problem 2. *Find the equivalent thermal conductivity between any two adjacent vertices joined by edges of thermal conductivity k in $\mathbb{Z}^m$ in the space $\mathbb{R}^m$. ($\mathbb{Z}^m$ is the set of m-tuples $(n_1, ..., n_m)$, where $n_1, .., n_m$ are integers).*
Note : Finding R_{eq} for some value(s) of m and stating there exists a pattern, would not fetch full marks.

Problem 3. *Take a cube of side $3a$, cut it into 27 $a \times a \times a$ cubes of equal volume, remove the middle cube. Now for each of the remaining cubes repeat this process ad infinitum. Let the final object have mass m . Find the moment of inertia about an axis through center perpendicular to a face of the cube.*